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The influence of the soil-foundation frequency-dependent behavior on the seismic performance of bridges

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Abstract

Bridge structure performance estimation under a seismic hazard can be significantly influenced by the level of modelling detail within the simulation of the soil structure interaction phenomenon. As the computational demands of such estimation greatly restricts the implementation of a detailed truncated FEM simulation of the semi-infinite soil domain, the Kelvin – Voigt Model within the framework of the substructure method along with other frequency independent simplified methods are frequently selected within the past literature as a computationally viable alternative. Due to their limited dynamic properties, the aforementioned simplified methods are prone to an inefficient approximation on the loading scenario of earthquake excitations with rich frequency content. To this end the limitations of the frequency independent simulation approach and subsequently the necessity of the frequency dependent simulation approach are thoroughly investigated in the following paper through the comparison of the seismic response of bridge structure under the use of the conventional Kelvin Voigt Models and the Lumped Parameter (LP) modelling method. Analyses results demonstrate a phenomenon of redistribution of damage among the bridge's structural components with the significance of error being highly correlated with the predominant frequency of the excitation.

Keywords: lumped parameter model, performance based analysis, soil structure interaction, bridge engineering

1 Introduction

Over the past decade, performance based analysis (PBA) has been firmly established in the field of damage assessment of structures, while various standardization documents have already implemented its conceptual framework in the procedures of

structural design. Broad application of the performance based Earthquake Engineering (PBEE) has been also observed in the field of bridge structures with a variety of studies published on the specific topic.

As the efficient estimation of a bridge structure performance can be only accomplished through the minimization of behavioural assumptions, a detailed simulation of each individual component of the bridge – soil system is considered essential. One direct approach on the SSI problem is the numerical solution of the semi-infinite soil – structure system with the use of the finite element method (FEM). The detailed truncated FEM simulation of the semi-infinite soil domain has been used in numerous occasions in the past [1]–[3]. However due to the nature of the FEM simulation of semi-infinite spaces some issues arise in regard to the computational viability of the method. As the modelling representation of the semi-infinite boundaries of the truncated model introduce the requirements of an extensive mesh refinement along with minimum dimensions of the soil domain [4], the FEM modelling approach cannot efficiently cope with the large amount of simulations needed for the implementation of the performance based analysis.

In an attempt to provide a computationally viable alternative to the FEM simulation, a large number of simplified SSI approaches have been proposed in the literature over the years such as the popular expansion of the substructure method in the time domain analysis [5]. According to the time domain substructure method, the ordinary differential equation system generated from the overall dynamic system's finite discretization is segmented in non-linear and viscous elastic (equivalent linear) regions, providing the capability of the appropriate dynamic condensation of the viscous elastic regions in the frequency domain. As a result the system's dimensions are reduced to the equations governing the nonlinear and interface regions, while the nonlinear behaviour essential to PBA is accurately included.

The time domain substructure method has been implemented in the simulation of bridge structures in multiple occasions [6]–[8], where the conventional Kelvin Voigt model, usually calibrated according to the predominant frequency of excitation, are selected as the representation of the semi – infinite soil domain in the condensed dynamic system. As the Kelvin-Voigt models are frequency independent, the scenario of inaccurate performance estimation cannot be easily dismissed. A frequency dependent behaviour was partially achieved by the addition of a mass component on the interface node of the Voigt Model in [9], [10], however limited efficiency was observed for complex soil-foundation dynamic behaviour.

In contrast to the Kelvin-Voigt limitations, the Lumped Parameter (LP) method is capable of accurately representing the frequency dependent characteristics of a viscous elastic dynamic system while maintaining a computational viability. A variety of approaches have been proposed in the past with differences in efficiency and applicability [11]–[15]. A complete LP modelling approach on the detailed SSI simulation of bridge structures has been proposed in [16], where the stability issues of past LP models have been effectively addressed. The proposed method has been also numerically verified with the direct FEM simulation of the structure-soil domain system through the case study of a concrete overpass bridge.

As the Kelvin-Voigt method is a broadly used approach in the analysis of bridge structures, it is considered essential to validate the method's limitations in comparison to the frequency dependent simulation through the LP modelling method. The current study investigates the influence of the frequency dependent foundation behaviour in the overall response of overpass bridges, through the comparison of the LP modelling procedure proposed in [16] and the conventional Kelvin-Voigt model method in the response of a reference bridge. The comparison is initiated through a general parametric study where the response divergence between the two methods relative to a number of different foundation and soil domain properties is thoroughly investigated. Thereafter the performance divergence between the two aforementioned methods is rigorously estimated both for each individual component and the overall bridge structure.

2 Selection and modelling details of the reference bridge

An overpass bridge along the Egnatia highway of Greece in the location of Pedini, whose dynamic response can be largely influenced by the adopted model of the foundations, is selected as a reference structure for the particular study. The bridge consists of a continuous three span deck of a total 71.2 m length, monolithically connected to two circular concrete piers of 9.1m height (Fig 1).

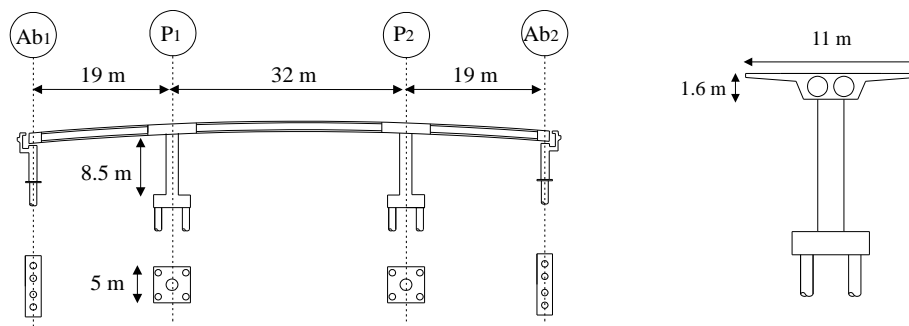


Figure 1. Structure selected as reference Bridge

The PTFE pot bearings are used to support decks at the abutments. Expansion joints are located in the longitudinal direction with 0.12 m of gap and shear keys are used in the transverse direction. The abutment foundation system consists of a four in line pile system while each pier is supported on a 2x2 pile-group. The soil properties of the bridge's site are defined by two cohesive soil layers, soft clay and medium density clay respectively with a dense limestone bedrock underlying the cohesive layers.

2.1 Soil Structure Interaction

The Soil structure interaction phenomenon is implemented in this study according to the proposed LP modelling procedure in [16]. According to the procedure the overall soil – structure dynamic system is initially represented as an ODE system according to a selected finite discretization (eq .1). The variables \mathbf{M} and \mathbf{C} denote the mass and damping matrices of the system, $\mathbf{f}(\mathbf{u})$ denotes the nonlinear restoring force which depends on the displacement vector, \mathbf{u} , and \mathbf{F}_i is the external loading vector applied to the respective degrees of freedom (DOFs).

$$\mathbf{M} \cdot \ddot{\mathbf{u}} + \mathbf{C} \cdot \dot{\mathbf{u}} + \mathbf{f}(\mathbf{u}) = \mathbf{F}_i \quad (1)$$

The dynamic system is divided into the non-linear segment (ss) consisting of the superstructure and the viscous elastic segment (vs) comprised by the abutments, foundation systems and the semi-infinite soil domain modelled through an equivalent linear approach. Interface regions (i) consist of multiple DOF connections located in the pier-foundations and the abutment–deck interfaces. The ODE system in Eq. (1) is dynamically condensed in the form of Eq. (2) through the appropriate calculation of the viscous elastic terms in the frequency domain. Fourier and inverse Fourier transformation operators are notated as \mathcal{F} and \mathcal{F}^{-1} respectively. The displacement vector \mathbf{u} in the frequency domain is denoted with the capital notation \mathbf{U} while the convolution operator is symbolized with the notation “ $*$ ”.

$$\mathbf{u} = \begin{bmatrix} \mathbf{u}_{ss} \\ \mathbf{u}_i \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} \mathbf{M}_{ss} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{ii}^{ss} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \mathbf{C}_{ss,ss} & \mathbf{C}_{ss,i} \\ \mathbf{C}_{i,ss} & \mathbf{C}_{ii}^{ss} \end{bmatrix}, \quad \mathbf{F}_i = \begin{bmatrix} \mathbf{0} \\ \mathcal{F}^{-1}(\mathbf{Vf}_{ve,sg} \cdot U_g) \end{bmatrix}$$

$$\mathbf{f}(\mathbf{u}) = \begin{bmatrix} \mathbf{f}_{s,ss}(\mathbf{u}_{ss}, \mathbf{u}_i) \\ \mathbf{f}_{s,ii}(\mathbf{u}_{ss}, \mathbf{u}_i) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathcal{F}^{-1}(\mathbf{S}_{ve,sg}) * \mathbf{u}_i \end{bmatrix}$$

Where $\mathbf{Vf}_{ve,sg} = -\mathbf{S}_{i,ve} \cdot \mathbf{S}_{ve,ve}^{-1} \cdot \mathbf{S}_b \cdot \mathbf{m}_b$, $\mathbf{S}_{ve,sg} = \mathbf{S}_{i,i}^{ve} - \mathbf{S}_{i,ve} \cdot \mathbf{S}_{ve,ve}^{-1} \cdot \mathbf{S}_{ve,i}$ (2)

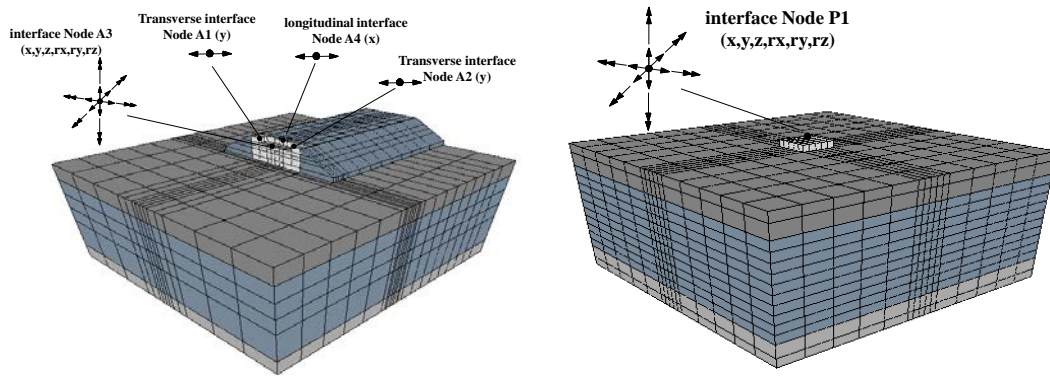


Figure 2 FEM models of a)abutment and b)pier base viscous segments in OpenSees

The dynamic properties of the viscous elastic segment are extracted in the form of the impedance function matrix $\mathbf{S}_{ve,sg}$ and the excitation compliance vector $\mathbf{Vf}_{ve,sg}$ from the FEM simulation of each interface region using the open source software OpenSees [17] (Fig. 2). The abutment interface region includes nine degrees of freedom (DOF) while the pier foundation region is limited to six DOFs. The impedance function matrix $\mathbf{S}_{ve,sg}$ is integrated on the condensed FEM model of the nonlinear segment (ss) through the appropriate dynamic spring assemblies as illustrated in Figure 4. The construction of the dynamic springs of each interface region assembly is accomplished according to the Kelvin – Voigt Model with parameters defined from the impedance function values in the predominant frequency of each excitation (approach 1) and the LP modeling assembly calibrated according to the extracted dynamic properties (approach 2).

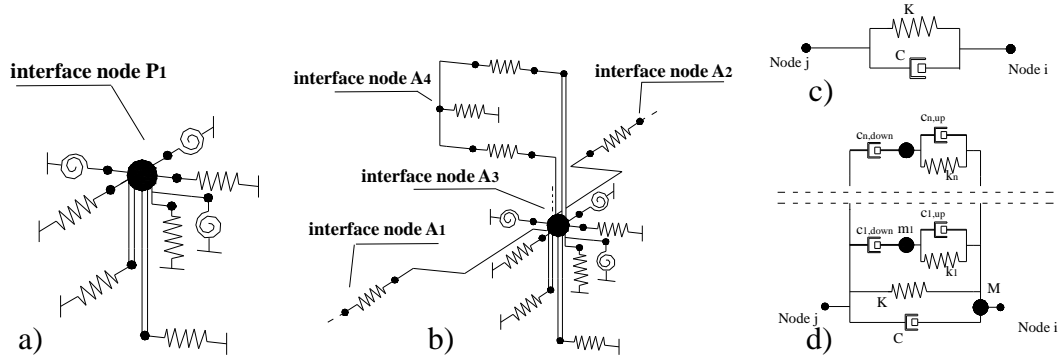
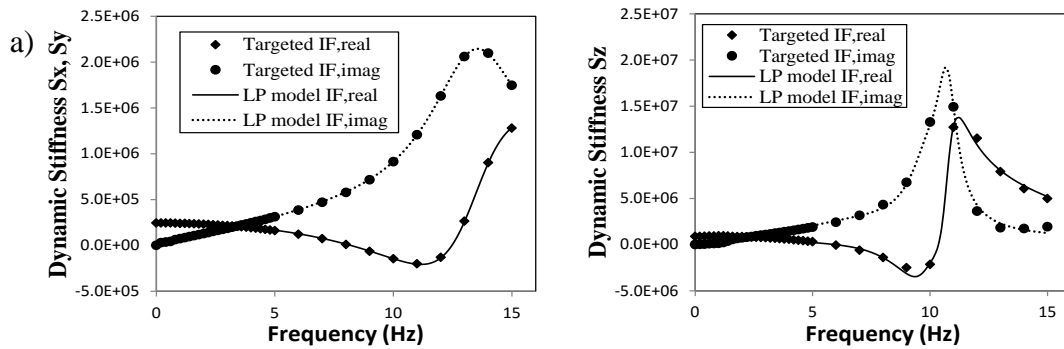


Figure 3. a) Dynamic spring Assembly on pier region b) Dynamic Spring Assembly of abutment interface region c) Dynamic spring as Kelvin Voigt Model d) Dynamic spring as LP model

The constrained optimization necessary for the calibration of the dynamic springs of each assembly is accomplished through the combined efforts of the interior point trust region algorithm [18] operating in a local level and a general multi-start stochastic algorithm managing the overall optimization scheme in a global level. A sample of the calibration results is illustrated in figure 5.



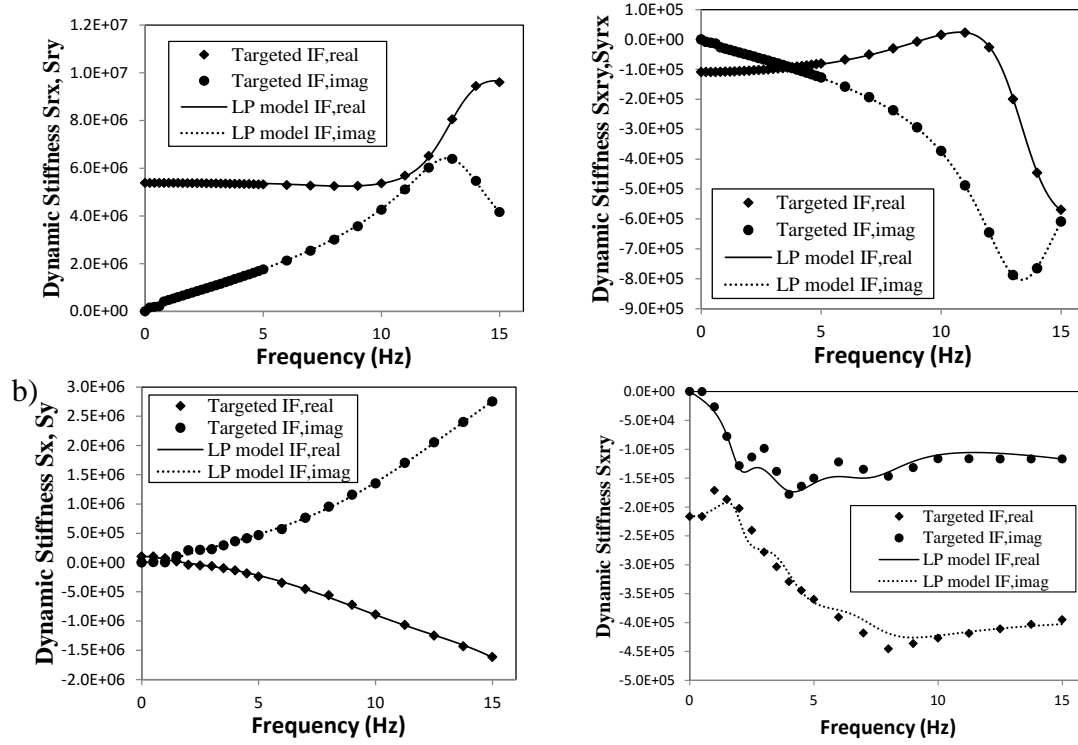


Figure 4. Sample of LP model Calibration results of a) Pier base and b) abutment interface region

The earthquake loading excitations are assigned in the condensed dynamic system in the form of a force vector applied on the systems interface DOF calculated from the convolution term $\mathcal{F}^{-1}(\mathbf{Vf}_{ve,sg} \cdot U_g)$.

2.2 Modelling assumptions of the nonlinear superstructure segment

The bridge superstructure segment is modelled through the finite element method in the open source software OpenSees. The deck and piers of the overpass bridge are modelled as beam finite elements. The bridge deck is simulated according to elastic material laws, as its prestressed nature is assumed to follow an uncracked behaviour. On the other hand fiber elements are used for the simulation of the reinforced concrete piers (fig. 5a), where the confined and unconfined concrete are modelled according to concrete material laws in [19], while a bilinear simplified stress strain behaviour is assumed for the reinforcement steel.

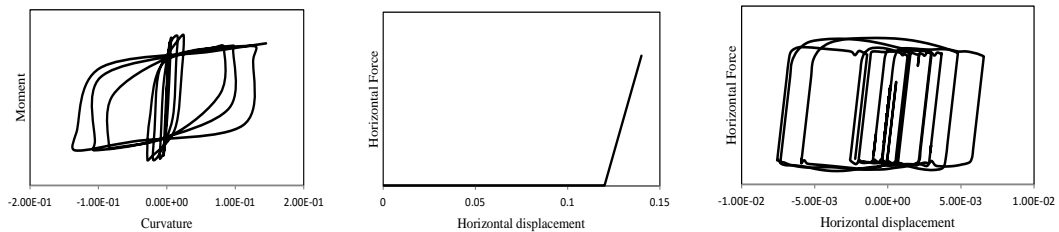


Figure 5. nonlinear behaviour a) of Pier Fiber section b)PTFE sliding bearing c) expansion joint

The PTFE sliding bearings located in the abutment to deck interface region, follow a velocity and pressure dependent friction model as proposed by Constantinou et al in [20] (fig 5.b). As the bridge pot bearings use a pure PTFE – stainless steel interface with lubrication, experimental data [21] on the type 1 Unfilled PTFE interface are used for the selection of the order of the exponential relation of the velocity to friction coefficient in Constantinou et al [20] .

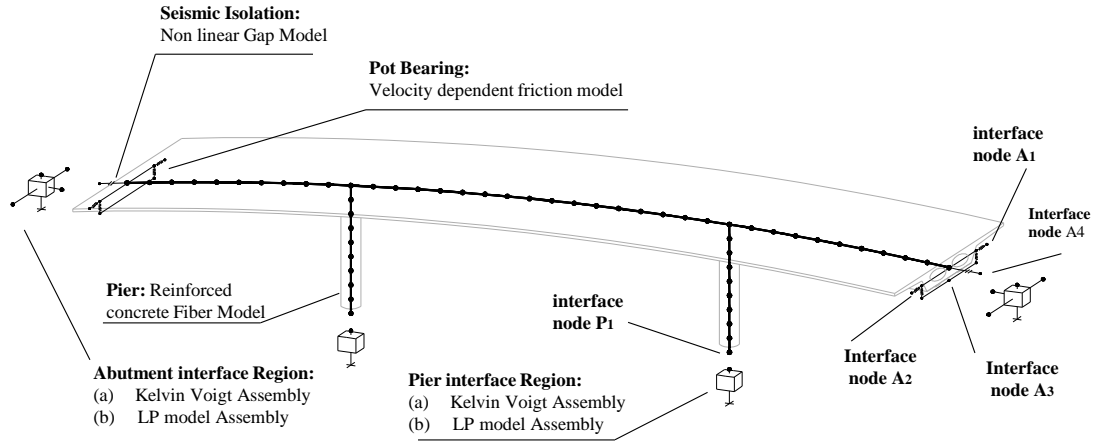


Figure 6. Construction of the Condensed Dynamic Model

The longitudinal expansion joint is simulated through the use of a gap element with an elastic behaviour after the gap closure according to rubber joint properties (fig. 5c). The overall model of the condensed dynamic system is illustrated in Figure 6.

3 Influence in the response of the bridge structure

3.1 Parametric analysis configuration

With the goal of defining the level of the bridge behaviour divergence between the two modelling approaches presented in the previous section and the expected Kelvin-Voigt approach error dependency on the properties of the soil foundation segment of the bridge, the reference bridge is parametrically studied in the following section. A group of six different homogeneous soil sites are selected in accordance to the need of the parametric analysis within the shear wave velocity $v_{s,30}$ range of 70 m/s-150 m/s. The soil properties of each site are approximated from their description based on [22](Table I).

Furthermore the difference in the 3x3 and 2x2 pile-group pier foundation system is studied in the parametric analysis while the abutment foundation system remains unaltered. A preliminary foundation design of each pile-group system to soil site combination was accomplished in the gravity loading combination according to

Eurocode 7, in order to obtain the realistic pier foundation dimensions for each soil scenario. Due to the large amount of different pile group foundation systems the impedance function matrices of the pier foundation interface region is approximated by the simplified approach proposed in [23], [24], while the abutment interface region impedance functions are generated from the previously introduced FEM model after the appropriate alterations.

Table I . Properties of the selected soil materials

ID	Soils	$\rho(\text{ton/m}^3)$	ν	$C_u(\text{MPa})$	$\Phi(^{\circ})$	$V_{s,30} \text{ (m/s)}$
S1	Very Soft Clay	1.3	0	10	-	73
S2	Very Loose Sand	1.7	0.34	-	29	80
S3	Soft Clay	1.5	0	30	-	97
S4	Medium Clay	1.8	0	50	-	120
S5	Loose Sand	2.1	0.35	-	30	129
S6	Loose Sand and Gravel	2.0	0.35	-	31	150

Twenty recorded ground motions from the PEER Strong Motion Database have been selected and included in the parametric analysis process. The ground motion selection was accomplished with the criteria of a mean response spectrum matching the design elastic response spectrum of Eurocode 8 for the seismic hazard of $\text{PGA}=0.16g$, while a limitation of 0.8-1.2 maximum scale factor was included as a restriction during the selection process.

3.2 Parametric analysis results

The differences in the dynamic response from the previously presented modelling approaches are measured through the value of the deck displacement in the separated loading scenarios of the two horizontal directions. The response divergence results are illustrated in figure 7.

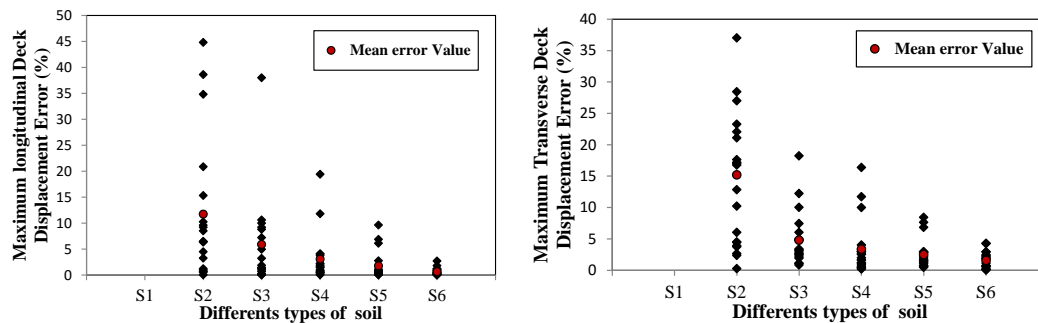


Figure 7a. Midpoint deck displacement for a 2x2 pile group foundation system in a a) longitudinal b) transverse excitation

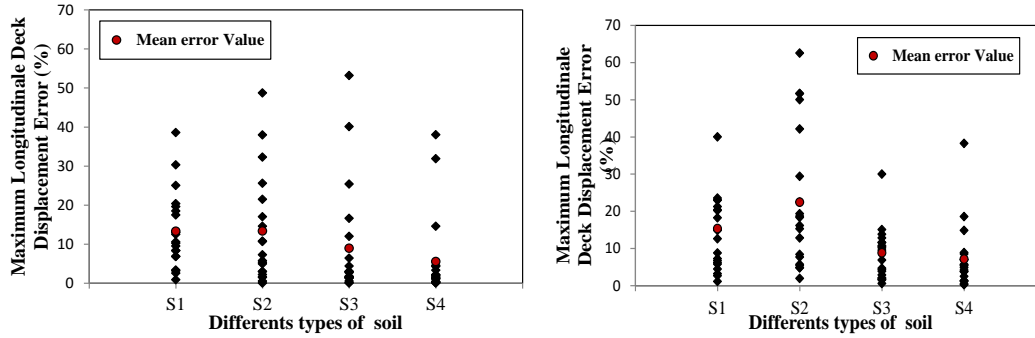


Figure 7b. Midpoint deck displacement for a 3x3 pile group foundation system in a) longitudinal
b) transverse excitation

It is observed that the divergence between the two methods is influenced by the complexity of the dynamic stiffness of the soil – foundation segment in the frequency domain as well as the Fourier amplitude distribution of the excitation. As the 2x2 pile-group follows an exponential increase in the irregularity of the impedance function matrix terms in relevance to the shear velocity of the soil domain, an analogous exponential relation in the divergence and soil flexibility is observed in the analysis results. On that scenario the behaviour divergence is observed to be linearly correlated with the predominant frequency of the excitation. On the other hand the complex 3x3 pile-group system with an impedance function irregularity distributed along the overall frequency region of most earthquake excitations follows a complex relation between soil flexibility and behaviour divergence, while the predominant frequency excitation is not anymore capable of representing the excitation frequency content as no correlation is observed between the predominant frequency and the behaviour divergence. It is also important to state that the behaviour divergence on the substructure method with Kelvin-Voigt models is not limited to soil foundation systems with irregular impedance functions in low frequencies, as high errors of the method were observed on stiffer soils for excitations with high frequency content.

4 Influence in the fragility of the bridge structure

The comparison of the two aforementioned time domain substructure methods in the performance level of a bridge structure is accomplished in the following section. As the frequency independent behaviour of the Kelvin Voigt model is expected to influence not only the overall displacement of the superstructure but also the distribution of stress among the components of the bridge it is essential for the particular study to focus on the damage of each individual component of the structure. Due to the computational needs of the analysis only the scenario of the original soil domain properties is included in this section.

4.1 Ground motion selection

As a strong correlation between the two modelling methods' divergence and the predominant frequency of the earthquake excitation has been observed in the previous section, ground motions are selected through a group division according to a verified indicator of the motion frequency content. The peak ground acceleration to peak ground velocity factor can be considered as such indicator as illustrated in [25]. Three groups of high, medium and low PGA/PGV ratio are created through the appropriate selection of 60 overall ground motions. Selection is limited to ground motions recorded on the surface of soils with shear velocity below 200m/s and an earthquake magnitude in the range of 5-7 M_w . Ground motions are scaled in the PGA range of 0.05g to 0.6g leading to a total of 720 total ground motions. The arbitrary amplitude scaling applied on the ground motion suite can lead to a false consideration of the appropriate dynamic properties of the earthquake motions but since the current study is focused on a method comparison, such miscalculations are considered negligible.

4.2 Selection of components limit states

Bridge components with inelastic behaviour are expected to develop different states of damage for different levels of intensity. For each component three different limit states are considered in the particular study, the Serviceability (LS1), the damage control (LS2) and the collapse prevention limit states.

Damage in pier components is developed through the form of plastic hinges on the corresponding ends of the pier. Since the moment redistribution of the bridge along with the axial forces in the pier due to the earthquake excitation are expected to vary in time, a macro level capacity limit state as the pier displacement can hide the actual performance of the specific components. Therefore limit states at the level of strain of each section at a plastic hinge location are implemented in the study. The yielding of the first reinforced bar in tension, initial crushing of the confined concrete and maximum confined concrete strain before hoop rupture correspond to LS1, LS2 and LS3 respectively.

Table III . Capacity limit states

Local Component ID	Serviceability (LS1)	Damage Control (LS2)	Collapse Prevention (LS3)	Calculation Method
Components 1 to 4: Plastic hinge locations on the Piers' ends	$e_s=0.0025$	$e_{cc}=0.00684$	$e_{cu} = 0.0104$	Fiber section strain
Component 5: Pot bearings	-	-	$u_b=17\text{cm}$	Bearing Geometry

e_s : Yielding of reinforcement steel (First yielding of Steel in tension)

e_{cc} : Confined Concrete strain of the compression region (Maximum Moment Capacity)

e_{cu} : Confined Concrete strain of the compression region (Hoop rupture)

The PTFE pot bearings located in the abutment to deck interface are expected to only develop minor degradation of the PTFE layer prior to significant lateral displacements. As a result no limit states are considered for the bearing components, with the exception of the collapse prevention limit state occurring with the unseating of the deck. As the abutments are modelled through an equivalent linear approach in the substructure method, the assumption of non-significant damage is included within the particular study.

4.3 Performance Analysis and results

The conditional probability of failure of an individual component for a specific level of intensity $P_{comp.}[D \geq C | IM]$ is calculated through a direct Monte Carlo method. The limit state probability of the complete overpass bridge can be obtained from the union of probabilities of each individual component in the same limit state as presented in [26].

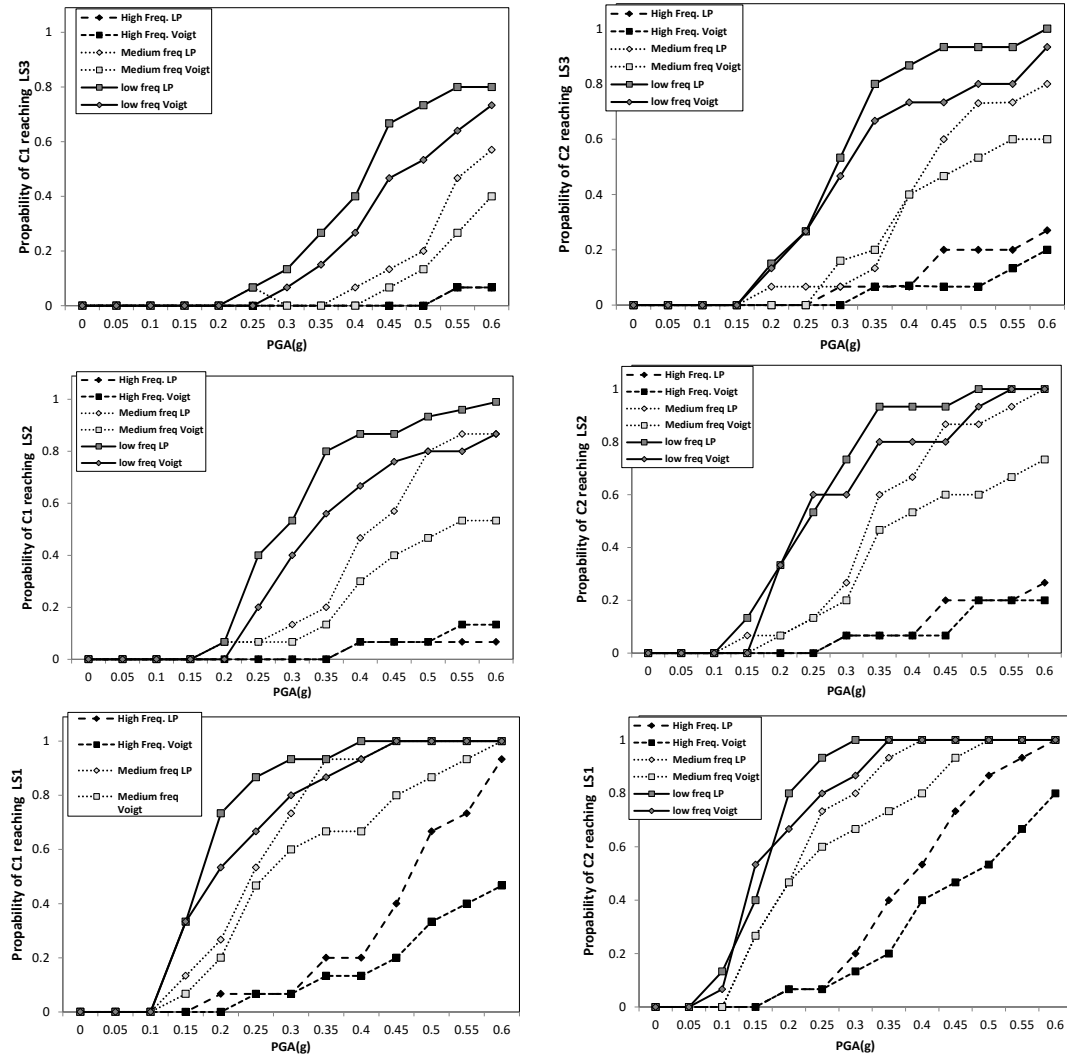


Figure 8. performance of pier components C1,C2 with excitation on the longitudinal direction

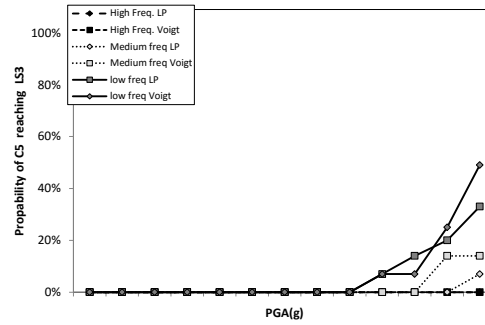


Figure 9. performance of pot bearing Component C5 in LS3

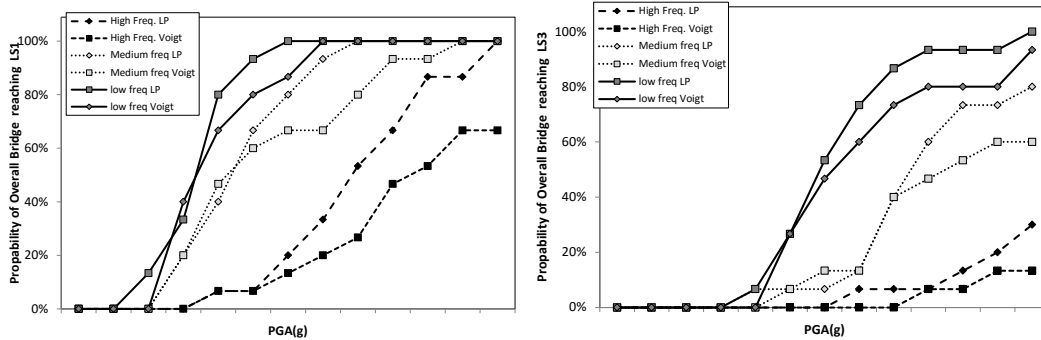


Figure 10. Performance of overall bridge for LS1 and LS3

The fragility curves of each individual component along with the fragility curves of the global overpass system are illustrated in the figures 8, 9 and 10. Components C1 and C2 represent the base and head of the first pier, while component 5 represents the pot bearings of the abutment. The second pier components are omitted as only slight differences appear in the results in comparison to the first pier as expected due to bridge's symmetry.

Through the comparison of the analysis results for the two SSI modelling approaches some concrete conclusions can be drawn. An important observation from the results of the performance analysis is the significant alteration of the sequence of local failure from the selection of the Kelvin Voigt method. The convex behaviour of the absolute value of the piers foundation impedance matrices in the frequency range 3-6 Hz, essentially contributes to the miscalculation of a more flexible soil domain for the neighbour frequencies, when the Kelvin Voigt model approximation is used. As a result the miscalculated flexibility of the pier foundations allows the bridge to close the gap with the abutment for a lower inertial force and as a result with a smaller level of damage developed in the plastic hinge locations of the piers. On this matter it is important to state that the selection of an equivalent linear behaviour for the abutment region is considered inefficient as further damage redistribution from the piers to the abutment back wall is shadowed from the simplified assumption. Furthermore earthquakes with higher frequency content tend to increase the Kelvin Voigt method's error due to a higher irregularity of I.F.s in higher frequency. Finally it is observed that as the stiffness of the system is

gradually reduced by the damage inflicted to its components the error is also gradually increasing.

5 Conclusions

In conclusion the efficiency of the frequency independent modelling approach of the substructure method through the Kelvin Voigt models is estimated through the comparison with the frequency dependent LP modelling method. The observed results suggest that under specific occasions frequency dependent modelling approach is mandatory. However it is important to state that the selection of the specific bridge structure highly influences the study towards the concluding results due to the high natural period of the bridge structure along with the significantly flexible soil domain. As a result further research on bridge structures of different conceptual design is essential.

Acknowledgements

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